Phase Transitions and Structural Relationships between Ge_5F_{12} , GeF_2 , SnF_2 , and TeO_2

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Oxidation of 20% of Ge(II) to Ge(IV) in GeF₂ leads to Ge₅F₁₂, whose chain structure with GeF₆ weak interchain bridges, is closely related to the chain structure of GeF₂ with weak interchain fluorine bridges. Results reported in the literature about a monoclinic high-temperature phase of GeF₂ are reexamined in the light of crystallographic and crystal chemistry considerations, which lead to the conclusion that the high-temperature cell is most likely erroneous. It is shown that GeF₂ most likely undergoes a ferro- to paraelastic $\beta \rightleftharpoons \gamma$ transition similar to the β -SnF₂ $\rightleftharpoons \gamma$ -SnF₂ and high-pressure TeO₂ \rightleftharpoons paratellurite TeO₂ transitions. High-temperature GeF₂ is probably tetragonal like γ -SnF₂ and TeO₂ and its powder pattern contains, in addition, weak peaks of Ge₅F₁₂ and/or SiO₂, due to hydrolysis of a minor part of the sample by moisture. @ 1989 Academic Press, Inc.

Introduction

Stannous fluoride SnF₂ is known to exist in three different crystalline phases, namely α , β , and γ (1). α -SnF₂, the stable phase at room temperature, crystallizes from $H_2O/$ HF solutions in the monoclinic C2/c space group. It contains two kinds of tin(II) atoms, Sn(1) in a pseudotetrahedral SnF_3E coordination and Sn(2) in a pseudooctahedral SnF_5E coordination, where E is the tin(II) electron lone pair (2). The two kinds of tin are linked together by fluorine bridges to form Sn₄F₈ tetramers. Upon heating to 150–190°C, depending on grain size (1, 3), the γ -phase is obtained. γ -SnF₂ crystallizes in the tetragonal space group $P4_12_12$ or $P4_{3}2_{1}2$, with tin(II) being in a SnF₄E trigonal bipyramid (4, 5). Upon cooling, at 66°C,

0022-4596/89 \$3.00 Copyright © 1989 by Academic Press, Inc. All rights of reproduction in any form reserved. γ -SnF₂ undergoes a second-order displacive transition, giving orthorhombic ferroelastic β -SnF₂ (space group $P2_12_12_1$) (1), which contains one kind of tin(II) in a distorted SnF₄F'E coordination intermediate between a trigonal bipyramid and an octahedron (4, 5).

GeF₂ crystallizes in the orthorhombic $P2_12_12_1$ space group and is isostructural to β -SnF₂ and high-pressure TeO₂ (6, 7). Ge(II) is coordinated by three fluorine atoms and a lone pair (*E*), forming GeF₃*E* pseudotetrahedra, linked together by weak interactions, while three-dimensional networks are observed in β - and γ -SnF₂. There is one report in the literature on the existence of a high-temperature form of GeF₂, which was found to be monoclinic (8). The preparation of a mixed oxidation state fluo-

ride of germanium, Ge_4F_{10} , better written as $Ge_3^{II}Ge^{IV}F_{10}$, has been published (9); however, its crystal structure shows that it is Ge_5F_{12} , i.e., $Ge_4^{II}Ge^{IV}F_{12}$, instead of Ge_4F_{10} (10). Mixed oxidation state fluorides of tin have also been reported, viz. Sn_7F_{16} , Sn_3F_8 , Sn_2F_6 , and $Sn_{10}F_{34}$ (11), but the crystal structure of Sn_3F_8 only is known (12), and they will not be discussed further here.

Stannous fluoride SnF₂ is stable in air at room temperature, except for a minor surface oxidation of the particles (13). Therefore, it can be easily studied and requires special precautions only at high temperature, when it should be held in a dry inert atmosphere to prevent hydrolysis to black SnO by moisture and oxidation to stannic tin by oxygen (1). On the other hand, GeF_2 is highly sensitive to air, as it is very hygroscopic, even deliquescent, resulting in orange germanous hydroxide being produced (6, 8-10, 14). This makes the study of GeF₂ much more delicate than that of SnF_2 . High-temperature studies are extremely difficult and hydrolysis in silica capillary tubes with attack of the glass have been reported (8). In contrast, Ge_5F_{12} , thought to be Ge_4F_{10} at the time, has been reported to be much less hygroscopic, and as a result, to be a convenient storage chemical for GeF_2 and GeF_4 , which are both extremely hygroscopic and are both obtained upon thermal decomposition of $Ge_5F_{12}(3)$.

We present here striking analogies between the crystal structures of GeF₂ and Ge₅F₁₂ on one hand and GeF₂, SnF₂, and TeO₂ on the other hand, which have not been reported before. The interrelationships between these structures led us to an analysis of the phase transition reported to take place for GeF₂ at 62°C. Analogies with the transition of TeO₂ at 9 kbar are also presented. The structural effect of the oxidation of 20% of Ge(II) in GeF₂ to Ge(IV), to give Ge₅F₁₂, is shown. Orthorhombic GeF₂ is referred to as β -GeF₂ hereafter because it is isotypic with β -SnF₂. In light of the structural interrelationships mentioned above, it is concluded that the high-temperature phase of GeF₂ is tetragonal, similar to that of γ -SnF₂ and paratellurite TeO₂, instead of monoclinic as reported in the literature.

Analogies between the Crystal Structures of GeF₂ and Ge₅F₁₂

1. Molecular Volume

The crystal structure of compounds with a stereoactive lone pair of electrons is usually highly distorted due to low symmetry at the central atom site (lone pair carrier), resulting in highly anisotropic structures. These are usually well interpreted in terms of bonding pair-nonbonding pair (lone pair) repulsions within the first sphere of coordination and between the first and second spheres of coordination (15–17). Galy et al. (16) showed that the volume occupied by the lone pair is equivalent to that of a fluorine or an oxygen atom in Te(IV) compounds. Denes et al. (2, 4) showed that this is also true for Sn(II) compounds. Although crystallographic data on germanium oxides/ fluorides are much more scarce, Table I shows that the above is also valid for Ge(II)-containing structures, as, with the exception of monoclinic GeF_2 , the V_2 molecular volume per anion, with the Ge(II) lone pair being counted as an anion, is fairly constant, regardless of the oxidation state of germanium. Table I also shows that the Ge(II) lone pair is stereoactive in orthorhombic GeF_2 and Ge_5F_{12} . The data for monoclinic GeF₂ will be discussed in the paragraph on the analogies between the crystal structures and phase transitions of GeF_2 , SnF_2 , and TeO_2 .

2. Unit-Cell Relationships

The unit-cell parameters for orthorhombic β -GeF₂ and Ge₅F₁₂ are given in Table II. As shown in Fig. 1, the monoclinic cell of

Compounds	Crystal system	V (Å ³) ^a	Z^b	V_1 (Å ³) ^c	$V_2 (\text{\AA}^3)^d$	References ^e
β-GeF ₂ ^f	Orthorhombic	201.51	4	25.19	16.79	(6)
GeF ₂	Monoclinic	283.55	4	35.44	23.63	(8), TW
GeF ₂	Monoclinic	283.55	5	28.36	18.90	(8), TW
GeF ₂	Monoclinic	283.55	6	23.63	15.75	(8), TW
γ-GeF ₂	Tetragonal	202.05	4	25.26	16.84	TW
Ge ₅ F ₁₂	Monoclinic	515.57	2	21.48	16.11	(10)
GeO ₂	Tetragonal	55.33	2	13.83	13.83	(18)
GeO ₂	Hexagonal	121.73	g	g	g	(19)

TABLE I Comparison of the Volume per Anion for Ge(II)- and Ge(IV)-Containing Fluorides and Oxides

^a Volume of the unit cell.

^b Number of molecules in the unit cell.

^c Molecular volume per anion (anions = F and/or O).

^d Molecular volume per anion (anions = F and/or O + E), counting the Ge(II) lone pair E as an anion.

e TW = this work.

^f Orthorhombic GeF₂ is called here β -GeF₂, because it is isostructural with β -SnF₂ (4).

^g As Z is not reported in (19), V_1 and V_2 could not be determined.

Ge₅F₁₂ is a bidimensional supercell of the orthorhombic cell of β -GeF₂. The matrix of transformation is

$$\begin{vmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{vmatrix}_{\text{Ge}_{5}\text{F}_{12}} = \begin{vmatrix} \frac{1}{2} & \frac{3}{2} & \mathbf{0} \\ \frac{3}{2} & -\frac{1}{2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{vmatrix} \cdot \begin{vmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{vmatrix}_{\boldsymbol{\beta} \cdot \text{GeF}_{2}} .$$
(1)

The result is that the relationship between the volumes is $V(Ge_5F_{12}) = 5V(\beta - GeF_2)/2$.



FIG. 1. Monoclinic cell of Ge_3F_{12} as a superlattice of the orthorhombic cell of β -GeF₂. Solid circles are Ge(II) atoms in β -GeF₂. Numbers 1 to 3 are Ge atoms in Ge₃F₁₂ according to the numbering in (10). Arrows indicate the shift of Ge atoms from GeF₂ to Ge₃F₁₂. Subscripts " β " and "5" refer to β -GeF₂ and Ge₃F₁₂, respectively.

This is in agreement with the molecular formula (GeF₂) \times 5/2 = Ge₅F₁₀, the two extra fluorine atoms resulting from the oxidation of 20% of Ge(II) to Ge(IV). As shown in Table II, the unit-cell parameters of the supercell (right-hand side column) are close to the observed values. The higher volume

TABLE II

Comparison of the Observed and Calculated Unit Cells of Ge_5F_{12} with That of β -GeF₂

·	$(\beta$ -GeF ₂) _o ^a	$(Ge_5F_{12})_o^a$	$(Ge_5F_{12})_c^b$
a (Å)	4.682	8.356	8.312
b (Å)	5.178	7,590	7.486
c (Å)	8.312	7.969	8.112
B	90	93.03	90.00
$V(\hat{A}^3)$	205.51	516.30	504.76
Z	4	2	2
Crystal system	Orthorhombic	Monoclinic	Monoclinic
Space group	$P2_{1}2_{1}2_{1}$	$P2_1/c^c$	$P2_{1}/c$
Reference	(6)	(10)	$\mathbf{T}\mathbf{W}^{d}$

^a Observed values.

^b Calculated values, as a supercell of β -GeF₂.

^c The structure of Ge_5F_{12} presented in (10) is described in the nonconventional space group $P2_1/a$. In this paper, all values have been converted to the conventional $P2_1/c$ space group (No. 14).

d TW = this work.

(2.4%) can be accounted for by the two extra fluorine atoms, although it is partly cancelled by the disappearance of the lone pair of 20% of the Ge(II) atoms, and maybe also by a more efficient packing. It is clear from these results that oxidation of one-fifth of the divalent germanium of β -GeF₂ to give Ge₅F₁₂ is, from a structural point of view, mostly a bidimensional phenomenon.

3. Structural Relationships

The structure of β -GeF₂ contains a pseudotetragonal pseudobody-centered sublattice of Ge similar to that observed in rutile-type GeO₂. The cationic sublattice is close to that of GeO₂, whereas comparison of Figs. 2a and 2b shows that the anion sublattice is much more perturbed, probably because of the fluorine lone pair repulsions. The doubling of the *c* axis of β -GeF₂ relative to that of GeO₂ results from the deviation from exact centering of Ge. A similar analogy of β -SnF₂ to SnO₂ was previously reported by us (4). The structure of



FIG. 2. Comparison of the structures of GeO₂ and β -GeF₂: (a) rutile-type GeO₂ with a tetragonal body-centered sublattice of Ge, octahedrally coordinated by oxygen; (b) distortion of the rutile lattice in β -GeF₂, with the *c* parameter being doubled. The dashed lines show longer Ge-F interactions. The supercell of Ge₃F₁₂ is shown.



FIG. 3. Comparison between the structures of β -GeF₂ and Ge₃F₁₂. Weak interactions are shown by dashed lines. (a) Chain structure of β -GeF₂ with weak interchain bridges. Ge-F distances are a = 1.788 Å, b = 1.909 Å, c = 2.094 Å, and d = 2.570 Å. (b) Pseudochain structure of Ge₃F₁₂ with weakly bridging Ge(IV)F₆ distorted octahedra.

 β -GeF₂ consists of zigzag chains of GeF₂ molecules linked by strong fluorine bridges along the **b** axis, with weaker bridges along **a**, making it essentially a chain structure (Fig. 3a).

In Ge₅F₁₂, the structure is more complex. As shown in Fig. 3b, a system of Ge(II)–F chains is still found parallel to the **b** axis; however, the bridges between each Ge₂F₄ group, within each chain, are weaker. In addition, in Ge₅F₁₂, the three-dimensional cohesion of the structure is due to weak bridges to the equatorial fluorines of the Ge(IV)F₆ distorted octahedra, the two axial fluorines of the octahedron being terminal.

The transition from β -GeF₂ to Ge₃F₁₂ upon oxidation of 20% of Ge(II) to Ge(IV) can be understood in terms of adding two fluorine atoms on Ge(2) to give a Ge(IV)F₆ distorted octahedron, with simultaneous motion of Ge(1)F₂ and Ge(3)F₂ groups, to form a new stable structure. The motion of the GeF₂ group, shown by arrows in Fig. 1, conserves the GeF₂ groups, except for Ge(2), which is oxidized to a Ge(IV)F₆²⁻ octahedron, and involves mostly a rearrangement of the lengths and angles of bridges.

TABLE IIIUNIT-CELL PARAMETERS AND ATOMICCOORDINATES FOR ORTHORHOMBIC β -GeF2, β -SnF2,AND HP-TeO2 AT ROOM TEMPERATURE

	β -GeF ₂ ^{<i>a</i>}	β -SnF ₂	HP-TeO ₂ ^b
a (Å)	4.682(1)	4.9889(7)	4.6053(6)
b (Å)	5.178(1)	5.1392(6)	4.8557(6)
c (Å)	8.312(1)	8.4777(14)	7.5300(10)
(x	0.266(4)	0.274(5)	0.274(2)
Ge/Sn/Tely	0.008(5)	0.023(7)	0.012(2)
z	0.131(3)	0.130(5)	0.117(2)
(x	0.528(24)	0.541(6)	0.549(2)
F(1)/O(1)	0.082(24)	0.163(5)	0.120(2)
z	0.982(18)	0.996(3)	0.940(1)
(x	0.433(18)	0.378(4)	0.406(2)
F(2)/O(2)	0.246(29)	0.202(2)	0.234(2)
z	0.279(18)	0.385(4)	0.333(1)
References	(6)	(4)	(7)

^a There are eight possible positions for the origin of the cell in $P2_12_12_1$. The structure of β -GeF₂ was solved using a different origin than β -SnF₂ and TeO₂. All results in this table are given using the same origin after the following transformation: $(x, y, z)_{GeF_2} = (\frac{1}{2} -$

x, y, z)_{TeO2/SnF2}.

^{*} At 19.8 kbar.

Analogies between the Crystal Structures and Phase Transitions of GeF_2 , SnF_2 , and TeO_2

1. Crystal Structures of β -GeF₂, β -SnF₂, and High-Pressure TeO₂

 β -GeF₂, β -SnF₂, and high-pressure TeO₂ (HP-TeO₂) crystallize in the $P2_12_12_1$ orthorhombic space group with similar unit-cell parameters and atomic coordinates (4, 6, 7), as shown in Table III. In all three cases, the cationic sublattice is pseudobody-centered, similar to that found in the rutile type (Fig. 2a). However, departure from the ideal body-centered lattice is shown in Fig. 2b by the lack of cations distant from c/2 in superimposing exactly on the projection parallel to **c**, which results in a doubling of the *c* parameter relative to rutile. Figure 2b also shows that the anionic sublattice is even more distorted compared



FIG. 4. Comparison between the structures of (a) HP-TeO₂ (Te-O distances are a = 1.912 Å, b = 2.034Å, c = 2.044 Å, d = 2.110 Å, and e = 2.586 Å) and (b) β -SnF₂ (Sn-F distances are a = 1.89 Å, b = 2.26 Å, c = 2.40 Å, d = 2.41 Å, e = 2.49 Å).

to rutile (Fig. 2a). Figures 3a and 4 also show the pseudobody-centered sublattice of cations in β -GeF₂, HP-TeO₂, and β -SnF₂.

Examination of Table IV shows that, despite their similar cells and atomic coordinates, the metal coordination is not the same in the three isotypic structures of β -GeF₂, HP-TeO₂, and β -SnF₂. Indeed, germanium in β -GeF₂ has a threefold coordination, with one short bond to a terminal fluorine and two longer bonds to bridging fluorines, which results in a chain structure, as shown in Fig. 3a. Weaker interactions between the terminal fluorines of one chain and the germaniums of neighboring chains

TABLE IV

The Six Shortest Metal-Anion Distances in Isotypic β -GeF₂, HP-TeO₂, and β -SnF₂

d_{M-X} (Å)	β-GeF ₂	HP-TeO ₂	β -SnF ₂	
1	1.79	1.912	1.89	
2	1.91	2.034	2.26	
3	2.09	2.044	2.40	
4	2.57	2.110	2.41	
5	3.26	2.586	2.49	
6	3.34	2.922	3.79	

provide sufficient cohesion to keep the compound in the solid state. For HP-TeO₂ (Fig. 4a), the tellurium coordination is 4, the weaker interaction observed in β -GeF₂ being much stronger and thus able to make a real oxygen bridge in HP-TeO₂. In addition, a fifth interaction, much weaker, is also observed. As a result, this is a threedimensional structure. In β -SnF₂ (Fig. 4b), tin is in a fivefold coordination similar to the tellurium coordination in HP-TeO₂, but with the fifth weak interaction of HP-TeO₂ being now strong enough to be a bridging bond. This is also a three-dimensional structure. The increase of coordination number from Ge(II) to Te(IV) and to Sn(II) is in agreement with increasing ionic radii: Ge²⁺, 0.736 Å; Te⁴⁺, 0.976 Å; Sn²⁺, 1.228 Å (20).

The coordination of Ge(II), Te(IV), and Sn(II) is governed by their size and electronic structure. All three have the same electronic structure, [Kr] $4d^{10}5s^25p^0$, with one lone pair of electrons nonengaged in bonding. However, this lone pair is hybridized: (i) sp^3 in Ge(II), resulting in a pseudotetrahedral GeF₃E coordination, (ii) sp^3d in Te(IV), giving a pseudotrigonal bipyramidal TeO₄E polyhedron, and (iii) sp^3d^2 in Sn(II), resulting in a distorted SnF_5E octahedron. In each case, one corner of the polyhedron is occupied by the lone pair E(16), and the polyhedron is highly distorted by the strong lone pair-bonding pair repulsions (15).

The bonding scheme and weaker interactions can be explained using the bond valence method of Brown (17), considering the various ways a regular octahedron can be distorted by a lone pair located on the central atom (Fig. 5). A lone pair along a threefold axis gives a distorted MX_3E tetrahedron; along a twofold axis, a MX_4E trigonal bipyramid; and along a fourfold axis, a distorted MX_5E octahedron, called by Brown \mathcal{C} , \mathcal{A} , and \mathcal{C} models, respectively. However, often the lone pair is not located



FIG. 5. Distortion of a regular octahedron by a lone pair of electrons along a rotation axis as defined by Brown (17). Model \mathcal{F} , as reported for HP-TeO₂, was hitherto unreported. Lone pair-bonding pair repulsions are shown by racminar; and lone pair-other atoms repulsions by \leftrightarrow .

along a rotation axis, and further distortion is observed as the polyhedron has no axial symmetry. This is the case of β -SnF₂, with a distorted SnF_5E octahedron (distorted \mathscr{C} model). In some cases, when the lone pair is located between two rotation axes, additional weaker interactions occur, such as the $GeF_3F'E$ polyhedron around Ge(II) in β -GeF₂, with three short bonds (although unequal), and a fourth weaker interaction. This is the *B* model of Brown, which is intermediate between a MX_3E tetrahedron and a MX_4E trigonal bipyramid. The $TeO_4O'E$ polyhedron in HP-TeO₂, with four strong bonds and a weaker interaction, is intermediate between a MX_4E trigonal bipyramid and a MX_5E octahedron. This model was not given by Brown (17), probably because no such geometry had so far been reported. We have represented it in Fig. 5 and called it F. The lone pair-bonding pair repulsions, larger than the repulsions between bonding pairs, in agreement with the VSEPR model (15), are responsible for the closing of X-M-X angles and are

shown by curved double arrows (\sim) in Fig. 5. The repulsions between the lone pair and the other atoms shown by straight double arrows (\leftrightarrow) are responsible for the smaller coordination numbers and apparent voids in the structures, which, in fact, are occupied by the lone pairs.

As stated earlier, a lone pair occupies a volume similar to that of a fluorine or an oxygen atom. Therefore, the molecular volume per anion V_2 (see Table I) should be similar for GeF₂ and GeO₂ on one hand and SnF_2 and SnO_2 on the other hand. Table I and the work of Denes et al. (2, 4) shows that this is the case. Discrepancies are due to packing efficiency of anions around the central cation (which is a function of the coordination number) and packing of polyhedra in the solid. Table V shows that V_2 for β -SnF₂ is 1.17% larger than that of SnO₂, whereas the value for β -GeF₂ is 21.40% larger than for GeO₂. This indicates that the fivefold coordination of Sn(II) in β - SnF_2 and the three-dimensional polymeric network of Sn-F bonds provides a packing almost as efficient as in rutile-type SnO₂. On the other hand, packing in β -GeF₂ is much more inefficient. This is probably due to the lower coordination number of Ge and to the pseudounidimensionality of this chain structure. The less dense packing of β -GeF₂ is also shown by the percentage dif-

TABLE V

Comparison of the Percentage Difference in Volume per Anion for MF_2 and MO_2 (M = Ge and Sn)

and the second	
$100 \times [V_2(\beta - \text{SnF}_2) - V_2(\text{SnO}_2)]/V_2(\text{SnO}_2) =$	1.17%
$100 \times [V_2(\beta - \text{GeF}_2) - V_2(\text{GeO}_2)]/V_2(\text{GeO}_2) =$	21.40%
$100 \times [R(Sn^{4+}) - R(Ge^{4+})]/R(Ge^{4+}) =$	30.19%
$100 \times [V_2(\text{SnO}_2) - V_2(\text{GeO}_2)]/V_2(\text{GeO}_2) =$	29.50%
$100 \times [R(Sn^{2+}) - R(Ge^{2+})]/R(Ge^{2+}) =$	66.85%
$100 \times [V_2(\beta - \mathrm{SnF}_2) - V_2(\beta - \mathrm{GeF}_2)]/V_2(\beta - \mathrm{GeF}_2) =$	7.87%

Note. The volumes of SnO₂ and GeO₂ (rutile form) are taken from the data of Ref. (21). The ionic radii of Sn⁴⁺ and Ge⁴⁺, in coordination 6, were taken from Ref. (20). The data for β -SnF₂ were taken from Ref. (4). β -GeF₂ refers to orthorhombic GeF₂ (6), as in Table I.

TABLE VI CALCULATED DENSITY VERSUS NUMBER OF

Molecules in the Unit Cell for m-GeF₂

	Z	$ ho_{\rm c}$ (g cm ⁻³)	$\Delta ho / ho ~ (\%)^a$
β -GeF ₂ ^b	4	3.644	
	٢4	2.59	-29.04
m-GeF ₂	25	3.22	-11.78
-	6	3.88	+ 6.30

 ${}^{a}\Delta\rho/\rho$ (%) = percentage density change at the β -GeF₂ \rightarrow m-GeF₂ phase transition.

^b Values taken from (6).

ference between the V_2 values of β -SnF₂ and GeF₂ of 7.87%, much lower than the 66.85% value predicted from the ionic radii of Sn²⁺ and Ge²⁺, whereas the value of 29.50% for SnO₂ and GeO₂ is very close to the theoretical value of 30.19% obtained from the ionic radii of Sn⁴⁺ and Ge⁴⁺. The larger metal-lone pair distance for Ge(II) [Ge(II)-E = 1.05 Å] than for Sn(II) [Sn(II)-E = 0.95 Å] (16) most likely results in more dead space in the β -GeF₂ structure, and, therefore, further decreases packing efficiency.

2. The Monoclinic Cell of GeF₂ at High-Temperature (m-GeF₂)

Hereafter, the high-temperature monoclinic phase of GeF₂ is referred to as m-GeF₂. Adams *et al.* (8) reported a phase transition in GeF₂ at 62°C, with $H_{\text{trans}}^{\circ} =$ -0.11 kcal mol⁻¹; therefore, this is a firstorder transition. The powder pattern of the high-temperature phase was indexed in monoclinic with a = 4.87 Å, b = 8.58 Å, c= 7.55 Å, $\beta = 116^{\circ}$ (we have changed the nonconventional cell of (8) to a conventional monoclinic cell with **b** as unique axis).

Using the volume of 283.5 Å³ for the cell of m-GeF₂, one can determine its calculated density (ρ_c) for given assumptions on the value of the number of molecules in the unit cell (Z). The results in Table VI show that no value of Z is likely to explain the β - $GeF_2 \rightarrow m$ -GeF₂ transition satisfactorily. Z = 4 results in a 29% decrease in compactness; such a high value is highly unlikely except for some transformations from high-pressure phases to ambient pressure phases. Z = 5 gives an 11.8% decrease in density. This is a reasonable value; however, it would be highly unlikely to have five molecules in the unit cell as this results in at least two different types of Ge (there is no site of multiplicity 5 in any monoclinic space group), and, therefore, the high-temperature phase would be more highly ordered than the low-temperature phase, a very unlikely situation. Z = 6 would result in a more highly packed and more highly ordered phase at high temperature than at room temperature, which are two highly unlikely situations.

Although automatic indexation using the procedure of Ito (22) is usually reliable and provides a reasonable indexation of the power pattern of GeF₂ at high temperature, the study of density changes shows that the monoclinic cell proposed in (8) is not adequate, as it would result in unlikely changes in packing. In addition, it is very rare to find a high-temperature phase of lower symmetry and more order than the room-temperature phase of the same compound.

3. New Propositions for the Modification of GeF₂ at High Temperature

The search for alternate explanations for the thermal and X-ray pattern changes of GeF₂ at high temperature are based (i) on the very high reactivity of GeF₂ and its sensitivity to traces of moisture, and (ii) on the structural analogies between GeF₂, SnF₂, and TeO₂. Even if the work in (8) was done carefully, it is difficult to be certain that no hydrolysis occurred. Two possibile reactions have been considered.

a. Partial hydrolysis giving a mixture of Ge_5F_{12} and GeO_2 . According to the reaction

$$6\text{GeF}_2 + 2\text{H}_2\text{O} \rightarrow \\ \text{Ge}_5\text{F}_{12} + \text{GeO}_2 + 2\text{H}_2 \quad (1)$$

one-sixth of the total GeF₂ is hydrolyzed by traces of moisture, giving GeO_2 and Ge_5F_{12} . The X-ray powder pattern of Ge_5F_{12} has not been published; however, it can be calculated from the unit cell given in (10). It is clear from the results of Table VII that the whole pattern of m-GeF₂ can be indexed in the monoclinic cell of Ge_5F_{12} . Moreover, the agreement between observed and calculated d spacings is better; as for the m-GeF₂ cell, the average $|d_0 - d_c|/d_0$ value is 0.51% with six values larger than 1%, the highest being 3.70%. On the other hand, for the Ge₅ F_{12} cell, the average value is 0.23%, only three are larger than 1%, and the highest value of 2.64% occurs for the first peak, i.e., at low θ angle, in a region where the d values are less accurate. To the contrary, in the m-GeF₂ cell, the largest value of 3.70%is found at a larger θ angle for the 10th observed peak. These results support well the hypothesis that Ge_5F_{12} could be the main component of high-temperature GeF₂. GeO_2 could be present as an amorphous or microcrystalline phase or in the rutile form. For the rutile-type phase, the (110), (200), and (211) peaks calculated from the data of (21) should occur at 3.1077, 2.1975, and 1.6197 Å, respectively, which agrees well with the peaks observed at 3.123, 2.191, and 1.630 Å, respectively.

b. Phase transition giving tetragonal γ -GeF₂, and partial hydrolysis producing HF, which attacks the silica capillary tubes. As shown above, orthorhombic β -GeF₂ is isotypic with β -SnF₂ and HP-TeO₂. The orthorhombic distortion of β -SnF₂ disappears at 66°C, giving tetragonal γ -SnF₂. Similarly, paratellurite TeO₂ is tetragonal at ambient pressure, and isotypic with γ -SnF₂. From these analogies, one can postulate that, at high temperature, GeF₂ gives a tetragonal form γ -GeF₂, isotypic to γ -SnF₂ and TeO₂. A good approximation of the

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TABLE VII

	m-0	GeF2 ^{<i>a</i>}			$\mathrm{Ge}_{5}\mathrm{F}_{12}{}^{b}$	
hkl	<i>d</i> ₀ (Å)	<i>d</i> _c (Å)	Δd (%)	h k l	$d_{\rm c}$ (Å)	Δd (%)
100	4.285	4.3739	2.07	200	4.172	2.64
020		4.2895	0.11			
110	4.008	3.8967	2.78	002	3.979	0.72
002	3.429	3.3952	0.99	021	3.425	0.12
1 1 2	3.310	3.2691	1.24	112	3.302	0.24
121	3.194	3.1816	0.39	121	3.194	0.00
				112	3.193	0.03
012	3.123	3.1569	-1.09	121	3.144	-0.67
101		3.1087	0.46			
022	2.672	2.6622	0.37	221	2.677	-0.19
031	2.606	2.6355	1.13	221	2.618	-0.46
				310	2.612	-0.23
201	2.416	2.4131	0.12	130	2.421	-0.21
				031	2.411	0.21
003	2.279	2.3634	3.70	131	2.306	-1.18
	<			222	2.256	1.01
102	2.227	2.2470	-0.90	312	2.235	-0.36
				302	2.225	0.09
032	2.191	2.1872	0.17	213	2.196	0.23
200		2.1870	0.18	321	2.184	0.32
040	2.144	2.1448	-0.04	321	2.136	0.37
				312	2.135	0.42
				032	2.135	0.42
023	1.987	2.0018	0.74	322	1.991	-0.20
122		1.9904	-0.17	004	1.989	-0.10
220	1.939	1.9484	-0.48	411	1.927	0.62
140		1.9257	0.69	014	1.924	0.77
104	1.877	1.8858	-0.47	330	1.872	0.27
2 1 1	1.818	1.8155	0.14	313	1.816	0.11
042		1.8132	0.26			
103	1.717	1.7251	-0.47	332	1.717	0.00
050		1.7158	0.07	214	1.714	0.17
				042	1.713	0.23
014	1.658	1.6653	-0.44	304	1.660	-0.12
051		1.6635	-0.33			
302	1.630	1.6221	0.48	422	1.633	-0.18
				510	1.630	0.00
151	1.613	1.6135	-0.03	511	1.613	0.00
				430	1.610	0.19
123	1.595	1.6005	-0.34	224	1.597	-0.13
				242	1.597	-0.13
				431	1.590	0.31
043	1.561	1.5568	0.27	034	1.564	-0.19
202		1.5543	0.43	015	1.558	0.19
222	1.459	1.4614	-0.16	2 4 3	1.462	0.21
300		1.4580	0.07	1 2 5	1.458	0.07
				324	1.458	0.07

Comparison of the Indexation of the Powder Pattern of the High-Temperature Phase of GeF₂ in the m-GeF₂ Monoclinic Cell (8) and in the Monoclinic Cell of Ge₃F₁₂ (10)

	m-(GeF2 ^a			${\rm Ge}_{5}{\rm F}_{12}{}^{b}$	
hkl	$d_{o}(\text{\AA})$	<i>d</i> _c (Å)	Δd (%)	hkl	<i>d</i> _c (Å)	Δd (%)
310	1.434	1.4374	-0.24	215	1.435	-0.07
				125	1.434	0.00
				243	1.433	0.07
320	1.381	1.3804	0.04	531	1.383	~0.14
				414	1.380	0.07
				424	1.378	0.22
053	1.364	1.3673	-0.24	610	1.368	0.30
232		1.3657	-0.12	611	1.360	0.29
				225	1.364	0.00
				144	1.363	0.07
143	1.341	1.3442	-0.24	334	1.340	0.07
015		1.3414	-0.03	135	1.339	0.15
				442	1.339	0.15
124	1.319	1.3226	-0.27	106	1.321	-0.15
				135	1.321	-0.15
				351	1.320	-0.08
				244	1.319	0.00
				053	1.318	0.08
330	1.298	1.2989	-0.07	106	1.299	-0.08
025		1.2947	0.25	621	1.299	-0.08
				523	1.297	0.08
				532	1.297	0.08
				153	1.296	0.15
251	1.258	1.2604	-0.19	443	1.259	-0.08
242		1.2586	-0.05	622	1.259	-0.08
054	1.209	1.2068	0.25	260	1.211	-0.17
				542	1.209	0.00
				054	1.207	0.17
				316	1.207	0.17
				451	1.207	0.17
352	1.182	1.787	0.28	524	1.184	-0.17
				226	1.183	-0.08
				245	1.183	-0.08
				452	1.183	-0.08
				542	1.182	0.00
$\Sigma \Delta d / N$	/(<i>hkl</i>)º: m-G Ge₅I	$eF_2 = 0.51\%$ $F_{12} = 0.23\%$	2			

TABLE VII—Continued

^a Data taken from (8) for monoclinic GeF_2 (m-GeF₂), after transformation to the standard monoclinic system with **b** as unique axis.

^b Calculated using the cell in (10), after transformation to the standard $P2_1/c$ space group, as given in Table II.

 $\Delta d = ((d_0 - d_c)/d_0) \times 100$. N(hkl) = number of (hkl) families of planes indexed.

unit cell of the hypothetic γ -GeF₂ can be obtained by analogy with the $\beta \rightarrow \gamma$ transition of SnF₂, and used for indexing its powder pattern. In addition, hydrolysis of a part of GeF₂ would most likely produce HF, which would attack the SiO₂ capillary tubes, producing SiF₄ vapor, which would hydrolyze back to SiO₂. We previously observed (23) that such mechanisms produce various varieties of silica. The data of Table VIII show that the powder pattern of GeF_2 at high temperature can be successfully indexed as a mixture of tetragonal GeF₂ and SiO₂ in α -quartz, α -cristobalite, and α -tridymite forms. The average relative difference between observed and calculated d_{hkl}

TABLE VIII

Comparison of the Indexation of the Powder Pattern of the High-Temperature Phase of GeF₂ in the m-GeF₂ Monoclinic Cell (8) and in a Mixture of Tetragonal γ -GeF₂ and SiO₂

		m-GeF	2ª		γ -GeF ₂ or SiO ₂ ^b			
Iobs	hkl	d _o (Å)	$d_{\rm c}$ (Å)	Δd (%)	Ι _γ	$(hkl)_{\gamma}/\mathrm{SiO}_{2}^{c}$	$d_{\rm c}$ (Å)	Δd (%)
vw	100	4.285	4.3739	-2.07		Q	4.260	0.58
	020		4.2895	-0.11				
mw	110	4.008	3.8967	2.78		С	4.040	-0.80
						Т	4.002	0.15
S	002	3.429	3.3952	0.99	S	110	3.434	-0.09
S	112	3.310	3.2691	1.24	VS	102	3.214	2.90
w	121	3.194	3.1816	0.39		Т	3.229	1.10
						С	3.162	1.00
S	012	3.123	3.1569	-1.09		С	3.138	-0.48
	101		3.1087	0.46				
w	022	2.672	2.6622	0.37		112	2.680	-0.30
m	031	2.606	2.6355	1.13	w	Т	2.591	0.58
w	201	2.416	2.4131	0.12	m	200	2.427	-0.46
w	003	2.279	2.3634	3.70	vw	201	2.335	-2.46
						Т	2.286	-0.31
vw	102	2.227	2.2470	-0.90	vw	113	2.197	1.35
w	032	2.191	2.1872	0.17	vw	210	2.170	0.96
	200		2.1870	0.18				
vw	040	2.144	2.1448	-0.04	w	004	2.145	-0.05
mS	023	1.987	2.0018	0.74	vw	104	1.962	1.26
me	122		1.9904	-0.17				
w	220	1.939	1.9484	-0.48	S	212	1.937	0.10
	140		1.9257	0.69				
mS	104	1.877	1.8858	-0.47	vw	203	1.850	1.44
						С, Т	1.870	0.37
mS	2.1.1	1.818	1.8155	0.14	m	1 1 4	1.819	-0.06
			1.8132	0.26				
m	103	1.717	1.7251	-0.47		220	1.716	0.06
	050		1.7158	0.07				
	014	1.658	1.6653	-0.44		Q	1.659	-0.06
mw	051		1.6635	0.33		-		
mw	302	1.630	1.6221	0.48	m	105	1.618	0.74
mw	123	1.595	1.6005	-0.34		204	1.607	-0.75
mS	043	1.561	1.5568	0.27		310	1.532	1.86
	202		1.5543	0.43				
$\Sigma \Delta d$	$ N(hkl)^d $	$m-GeF_2 =$	0.69					
1 14		γ -GeF ₂ or	$SiO_2 = 0.78$	3				

a,b,d As in Table VII.

^c Q = α -quartz, C = α -cristobalite, T = α -tridymite.

is only slightly larger than for the m-GeF₂ cell and the largest discrepancy is smaller (2.90 for tetragonal GeF₂, 3.70 for m-GeF₂). In addition, the peak intensity pattern for tetragonal GeF₂, hereafter called γ -GeF₂ by analogy with isotypic γ -SnF₂, is similar to that of γ -SnF₂, as expected. Except for one, all peaks for SiO₂ are weak, therefore, due to minor contamination.

4. Orthorhombic \rightarrow Tetragonal Phase Transitions of GeF₂, SnF₂, and TeO₂

The isotypic orthorhombic phases of β -GeF₂, β -SnF₂, and HP-TeO₂ belong to the $P2_12_12_1$ space group, which can exhibit the property of "ferroelasticity"; i.e., the crystals are subject to an internal spontaneous strain, which usually results in multidomain crystals. Existence of ferroelastic behavior has been proved in β -SnF₂ and HP-TeO₂, using the techniques of X-ray and neutron powder diffractions (4, 5, 7, 24). These compounds undergo a phase transition from ferroelastic to paraelastic properties, upon heating, at 66°C for SnF₂, and upon reducing pressure, at 9 kbar for TeO₂. In both cases, the phase transition is a secondorder displacive one, such that the tetragonal space group of the paraelastic phase is a supergroup of $P2_12_12_1$, in agreement with Landau's theory of second-order transitions (25), and as tabulated by Ascher (26)and Aizu (27). Both paraelastic γ -SnF₂ and TeO_2 crystallize in $P4_12_12$ or its enantiomorph $P4_{3}2_{1}2$. As β -GeF₂ is isotypic to HP-TeO₂ and β -SnF₂, one could expect it to experience a similar type of phase transition upon heating, giving a high-temperature tetragonal phase of GeF₂ that we call γ -GeF₂ by analogy with γ -SnF₂. The likelihood of such transition taking place in GeF₂ is reinforced by the two following observations:

(i) All strong diffraction peaks of HT-GeF₂ can be indexed in a tetragonal cell similar to that of γ -SnF₂, as shown in Table VIII. In addition, the similar pattern of line intensities suggest that γ -SnF₂ and γ -GeF₂ are isotypic. The weak lines can be explained by hydrolysis of a part of the sample, giving SiO₂ and/or Ge₅F₁₂.

(ii) The following mathematical relationships between the unit-cell parameters of β -GeF₂ and m-GeF₂ suggest that the cell of m-GeF₂ is a superstructure of that of β -GeF₂.

$$a_{\rm m} \approx (a_{\beta} + b_{\beta})/2$$
 (2)

$$b_{\rm m} \approx c_{\beta}$$
 (3)

$$c_{\rm m} \approx [(3a_{\beta}/2)^2 + (3b_{\beta}/4)^2]^{1/2}$$
 (4)

$$\beta \approx 90^{\circ} + \arcsin[(3a_{\rm m}/4)/c_{\rm m}],$$
 (5)

where $a_{\rm m}$, $b_{\rm m}$, $c_{\rm m}$, β , and a_{β} , b_{β} , c_{β} are the unit-cell parameters of m-GeF₂ and β -GeF₂, respectively.

The tetragonal unit cell for γ -GeF₂, obtained from the indexation of Table VIII, is comparable to the cells of isotypic γ -SnF₂ and paratellurite TeO₂ (Table IX). Contrary to the monoclinic cell, it requires no drastic change of density. It follows that the β -GeF₂ cell is an orthorhombic distortion of the γ -GeF₂ cell, and that the m-GeF₂ cell is a superstructure of that of γ -GeF₂, the two cells being related as follows:

$$\begin{vmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{vmatrix}_{\gamma} = \begin{vmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{2}{3} \\ 0 & 1 & 0 \end{vmatrix} \begin{vmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{vmatrix}_{m}$$
(6)

$$V_{\gamma} = 2V_{\rm m}/3. \tag{7}$$

The relationship between the monoclinic and the tetragonal cells of $HT-GeF_2$ is shown in Fig. 6.

As stated above, the ferro- to paraelastic transitions of TeO_2 and SnO_2 are second-order transition; i.e., they do not involve any latent heat and volume change at the transition point. However, the first derivatives of these parameters, i.e., the heat capacity and thermal expansion coefficients (versus T) or hydrostatic compressibility

	TABL	E IX				
Unit Cell of Paraelastic Paratellurite TeO_2 , γ -SnF ₂ , and γ -GeF ₂						
	TeO ₂	γ-SnF ₂	γ-GeF ₂			
a (Å)	4.8052	5.072	4.853			
c (Å)	7.6021	8.493	8.579			
V (Å ³)	175.53	218.48	202.05			
Ζ	4	4	4			
$\rho_{\rm c} ({\rm g} {\rm cm}^{-3})$	6.04	4.76	3.64			
T (°C)	20	80	80			
Space group	$P4_{1}2_{1}2$ or	P4 ₁ 2 ₁ 2 or	P41212 or			
-	P4 ₃ 2 ₁ 2	P43212	P43212			
Reference	(7)	(28)	$\mathbf{T}\mathbf{W}^{a}$			

 a TW = this work.

coefficients (versus P), can undergo drastic changes, and this has been shown to be the case for the expansion coefficients of SnF₂ (1, 5, 28) and TeO₂ (7, 24). However, a DTA study of GeF_2 has shown that the transition at 62°C is associated with an enthalpy of -0.11 kcal mol⁻¹. Therefore, this is a first-order transition. As a consequence, there is no subgroup-supergroup requirement for the space group of γ -GeF₂ as for TeO₂ and γ -SnF₂. Thus, in principle γ -GeF₂ need not crystallize in the P4₁2₁2 or $P4_{3}2_{1}2$ space group. However, of course, it can, and the indexation of Table VIII and the similar intensity of Bragg peaks suggest that it does. It is well known that a phase transition that is of second order at a given pressure can be of first order at another pressure. Therefore, there is no contradiction in the fact that the $\beta \rightleftharpoons \gamma$ -SnF₂ transition is a second-order one and the $\beta \rightleftharpoons \gamma$ -



FIG. 6. γ -GeF₂ tetragonal cell and m-GeF₂ supercell.



FIG. 7. Unit cell parameters and volume versus temperature, as observed for $\beta \rightleftharpoons \gamma$ -SnF₂, and as suggested for $\beta \rightleftharpoons \gamma$ -GeF₂. (Scales are in arbitrary units.)

GeF₂ a first-order one. In addition, the reversibility of the $\beta \rightleftharpoons \gamma$ -GeF₂ transition, not required for first-order transitions, suggests that it can be second order at another pressure and that it requires no drastic structural change. Therefore, it is probably displacive, like the transition of SnF₂. Most likely, a sudden change of unit-cell parameters and volume occurs at the transition, of the type suggested in the model of Fig. 7.

Conclusion

The crystal structures of Ge_3F_{12} , GeF_2 , SnF₂, and TeO₂ are closely related. The unit cell of Ge_5F_{12} is a supercell of that of β -GeF₂. The chain structure of β -GeF₂, with weak interchain links through secondary interactions, changes to a similar chain structure with weakly bridging GeF₆ units between the (GeF₂)_n chains, upon oxidation of 20% of Ge(II) to Ge(IV).

The data reported for the high-temperature phase of GeF₂ have been reanalyzed. Based on crystallographic considerations and the structural relationships quoted above, it is concluded that the monoclinic cell reported in the literature is a supercell of the real cell, which is most likely tetragonal, isotypic to γ -SnF₂ and TeO₂ paratellurite. The powder pattern of HT-GeF₂ can be successfully indexed in the monoclinic cell of Ge_5F_{12} or in a tetragonal γ -GeF₂ cell with SiO₂ impurity lines, which could be due to hydrolysis of a part of the sample by traces of moisture. The tetragonal γ -GeF₂ cell, similar to that of γ -SnF₂ and TeO₂, accounts well for molecular volume considerations (Table I) and number of molecules in the unit cell (Table VI), contrary to the monoclinic supercell. In addition, it fits a model of ferro- to paraelastic phase transitions which is observed in SnF₂ and TeO₂.

Upon heating GeF₂ in the experiment of (8), hydrolysis of a part of the sample probably occurred, resulting in some Ge₅F₁₂ and/or SiO₂ being produced. This does not contradict the reversibility reported in (8), as the weak peaks of Ge₅F₁₂ may be difficult to see at room temperature because of overlapping with the peaks of γ -GeF₂, which is probably responsible for the line broadening reported in (8). The reversibility observed in (8) was probably the reversible first-order transition $\beta \rightleftharpoons \gamma$ -GeF₂, all strong peaks of HT-GeF₂ being concerned by this transition.

A study of GeF_2 versus temperature, by X-ray diffraction on a single crystal or by neutron diffraction on powder, would be most appropriate to study the atomic displacement as a function of temperature in order to confirm the proposed ferroelastic properties.

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